Q1a

a)

SIGN CHANGE SUGGESTS ROOT IN INTERVAL $0.9 < x < 3.4$
THERE ARE THREE ROOTS IN INTERVAL SO MAY NOT CONVERGE ON 0.98, INTERVAL IS TOO BIG

Q1b

b) USE BOUNDS

LB 0.975

UB 0.985

$f(0.975) = -0.0462\ldots$

$f(0.985) = 0.0210\ldots$

SIGN CHANGE

OR OTHER SUITABLE VALUES

$f(0.97) = -0.7933\ldots$

$f(0.99) = 0.05509\ldots$

SIGN CHANGE

GRAPH IS CONTINUOUS THROUGH THE INTERVAL $0.975 < x < 0.985$ (OR $0.97 < x < 0.99$)
SIGN CHANGE INDICATES ROOT CLOSE TO 0.98

Q1c

c) LB 0.9815 UB 0.9825

$f(0.9815) = -0.002665...$

$f(0.9825) = 0.004081...$

SIGN CHANGE AND CONTINUOUS FUNCTION WITHIN INTERVAL $0.9815 < x < 0.9825$ ALL VALUES ROUND TO 0.982 SO $f(x)=0$ IS 0.982 3SF

Q2a

a) STAIRCASE DIAGRAM

$x_0 = 0.5$

Q2b

b) i) $x_0 = 0.5$

$x_1 = 0.90856\ldots$

$x_2 = 1.48783\ldots$

$x_3 = 2.05012\ldots$

$\boxed{x_1 = 0.909, \quad x_2 = 1.49, \quad x_3 = 2.05}$

ii) MANY MORE ITERATIONS NEEDED AS SHOWN IN DIAGRAM FROM PART (a)

STILL FAR FROM 3.5 SUGGESTING CONVERGENCE IS SLOW

Q2c

c) LB 3.485 UB 3.495

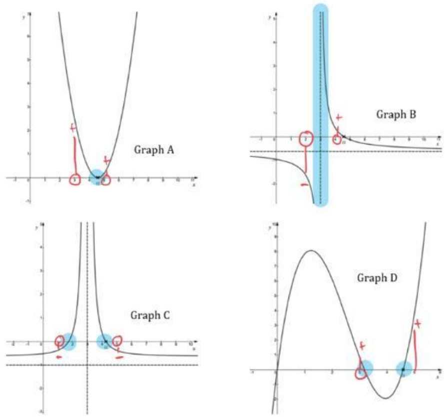
$f(x) = f(3.485) = -0.07956\ldots$

$f(3.495) = 0.05643\ldots$

SIGN CHANGE AND CONTINUOUS FUNCTION THROUGH INTERVAL $3.485 < x < 3.495$ ALL VALUES ROUND TO 3.49 TO 3SF SO $f(x)=0$ IS 3.49 3SF

Q3

The diagrams below show the graphs of four different functions.



Match each graph above with the correct statement below.

1) GRAPH B

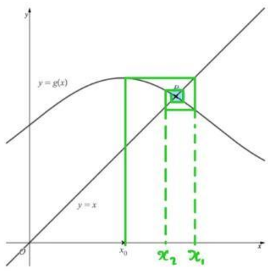
2) GRAPH C (OR D)

3) GRAPH D

4) GRAPH A

Q4a

The diagram below shows the graphs of $y = x$ and $y = g(x)$.



a) DRAW UP FROM x_0 TO $g(x)$ THEN ACROSS TO STRAIGHT LINE $y=x$ AND REPEATE

Q4b

b) $x - \sin 0.8x = 2.5$

$x = g(x)$

REARRANGE

$x = \sin 0.8x + 2.5$

$x_{n+1} = \sin 0.8x_n + 2.5$

$x_0 = 2$

$x_1 = 3.4995....$

$x_2 = 2.8353...$

$x_3 = 3.2664...$

$x_4 = 3.00415...$

$x_5 = 3.17300...$

$x_6 = 3.0672...$

$x_7 = 3.134....$

$\boxed{x = 3.1 \quad 2sf}$

Q4c

c) LB $x = 3.1$ UB

3.05 3.15

$f(x) = 0$

$f(x) = x - \sin 0.8x - 2.5 = 0$

$f(3.05) = -0.09543...$

$f(3.15) = 0.06766...$

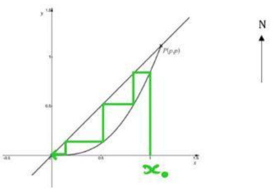
CONTINUOUS FUNCTION AND SIGN CHANGE ACROSS THE INTERVAL $3.05 < x < 3.15$

ALL NUMBERS IN INTERVAL ROUND TO 3.1

SO $f(x) = 0$ IS 3.1 TO 2SF

Q5a

The village of Crinkley Bottom lies on a straight road, as modelled by the line $y = x$ on the graph below. Rush hour traffic causes much air pollution in the village so to improve the air quality around Crinkley Bottom a bypass is to be built.
The path of the bypass is modelled by part of the equation $y = x^2 \sin x$.



a) STAIRCASE / LADDER FROM ANY VALUE x_0 WHERE $0 < x_0 < p$

Q5b

b)

$x_0 = 1$

$x_1 = 1.090135...$

$x_2 = 1.108803...$

$x_3 = 1.1129495...$

$x_4 = 1.113884...$

$x_5 = 1.114095...$

$P = 1.114$ 4sf

$$\boxed{P = (1.114, 1.114) \quad (4sf)}$$

Q5c

c) $P = 1.114$

REARRANGE $x = g(x)$ INTO $f(x) = 0$

$f(x) = x - x^2 \sin x$

$f(1.1135) = 0.001016...$

$f(1.1145) = -0.000530...$

SIGN CHANGE OVER CONTINUOUS INTERVAL
$1.1135 < x < 1.1145$ MEANS $f(x) = 0$
HAS ROOT OF 1.114 (4 SF)

ALTERNATIVE REARRANGEMENTS OF $x = g(x)$
MAY GIVE DIFFERENT VALUES BUT SIGN
CHANGE WOULD STILL SHOW ROOT

Q6a

a) $x = g(x)$

$$x_{n+1} = 3 - \ln(x_n + 1)^2$$

$$x_0 = 1.8$$

$$x_1 = 1.93988\ldots$$

$$x_2 = 1.837117\ldots$$

$$\vdots$$

$$x = 1.88 \text{ (3sf)}$$

COULD STOP AT $x = 1.9$ (2sf) OR CONTINUE TO $x = 1.88$ (3sf)

X-COORDINATE FORMS ISOSCELES TRIANGLE

$$\sqrt{1.88^2 + 1.88^2} \text{ OR } \sqrt{2} \times 1.88$$

$$2.65$$

$$\boxed{2.7 \text{ (2sf)}}$$

Q6b

b) $x = g(x)$

$$x_{n+1} = 4 - \ln(x_n + 1)^2$$

$x_0 = 2.5$

$x_1 = 2.4305\ldots$

$x_2 = 2.4803\ldots$

$\vdots$

$x = 2.46$ (3sf)

COULD STOP AT $x = 2.5$ (2sf) OR CONTINUE TO $x = 2.46$ (3sf)

X-COORDINATE FORMS ISOSCELES TRIANGLE

$\sqrt{2.46^2 \times 2}$ OR $\sqrt{2} \times 2.46$

3.47

$\boxed{3.5 \text{ (2sf)}}$

Q7a

a) $20e^{-0.15t} - 0.2t = 0$

$20e^{-0.15t} = 0.2t$

$e^{-0.15t} = 0.01t$

$0.01t = \frac{t}{100}$

$-0.15t = \ln\left(\frac{t}{100}\right)$

$0.15 = \frac{3}{20}$

$-\frac{3}{20}t = \ln\left(\frac{t}{100}\right)$

$\frac{3}{20}t = -\ln\left(\frac{t}{100}\right)$

$\ln\left(\frac{t}{100}\right)^{-1}$

$\frac{3}{20}t = \ln\left(\frac{100}{t}\right)$

$$\boxed{t = \frac{20}{3}\ln\left(\frac{100}{t}\right)}$$

Q7B

b)

$t_0 = 12$

$t_1 = 14.135..$

$t_2 = 13.043...$

$t_3 = 13.579...$

$t_4 = 13.310...$

$t_5 = 13.443...$

$t_6 = 13.377...$

$$\boxed{t = 13.4 \text{ SECONDS } (3sf)}$$

Q7C

c)

$13.4 \times 100000 = 1340000$ SECONDS

$\div 60$ MINS

$\div 60$ HOURS

$\div 24$ DAYS

$= 15.5092...$

$$\boxed{16 \text{ DAYS}}$$